SCATTERING IN CHERN-SIMONS MATTER THEORIES

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INTRODUCTION

CHERN-SIMONS THEORY: WHAT

The **Chern-Simons action** in $(2+1)$ dimensions is

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$\mathcal{L}_{\mathrm{CS}}$ is NOT gauge invariant. But if the **level** $k \in \mathbb{Z}$ then we note that

$$\exp(i S_{\mathrm{CS}}[A])$$

is unchanged. This is all we care about for our quantum theory.

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Some neat facts about Chern-Simons theories:

- Time reversal invariant depending on level of k and gauge group
- Bosonization duality between Chern-Simons with bosons and with fermions
- Large N Chern-Simons matter theories are dual to higher spin gravity in AdS_4

Chern-Simons with $U(N)$ gauge fields coupled to fundamental bosons and fermions

$$\mathcal{L}_{\text{Bos}} = \mathcal{L}_{\text{CS}} + D_\mu \bar{\phi} D^\mu \phi + m_B^2 \bar{\phi}\phi + \frac{b_4}{2N_b}(\bar{\phi}\phi)^2$$

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Supported by evidence:

- 3pt functions match in $N \to \infty$
- Thermal partition functions matching in $N \to \infty$
- SUSY duality can be constructed which flows to these ones after SUSY breaking

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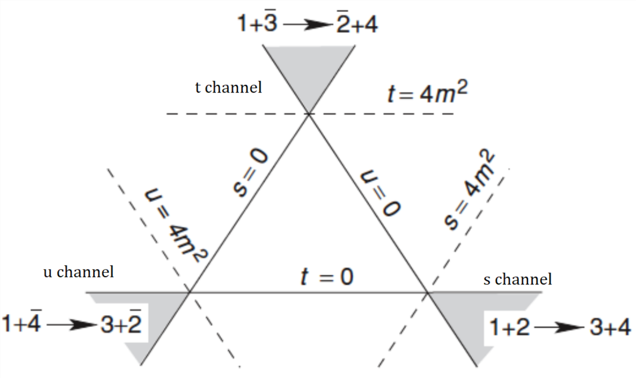
Causality is usually upgraded to a slightly stronger condition: that transition amplitudes are real-boundary values of analytic functions $F(s, t, u)$ in complex s-plane.

$$S = \mathbb{1} + i\,(2\pi)^d\,\delta^{(d)}(p_1 + p_2 - p_3 - p_4)\;F(s, t, u)$$



Crossing symmetry means the amplitude for something like

$$s : A_1 + A_2 \to A_3 + A_4$$

can be obtained from

$$u : A_1 + \bar{A}_4 \to A_3 + \bar{A}_2$$

by analytic continuation of $F(s, t, u)$.

THE PROJECT

LITERATURE: DUALITY OF AMPLITUDES

In (1404.6373), Jain, Mandlik, Minwalla, Takimi, Wadia, Yokoyama calculated amplitudes to test the boson-fermion fundamental matter duality.

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- Unusual properties including **failure of crossing symmetry**. $\psi\psi \to \psi\psi$ is modified by

$$f(\lambda) = \frac{\sin(\pi\lambda)}{\pi\lambda}$$

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- Believe this factor comes from a need to attach Wilson lines to fermions to make correlation functions gauge invariant.
- Not clear if what they found is a large N artifact. Want to do calculation in usual perturbative regime ($k \to \infty$) but N finite.

OUR RESEARCH: FERMIONS AT ONE-LOOP

We calculate fundamental fermion scattering at one loop to test crossing symmetry.

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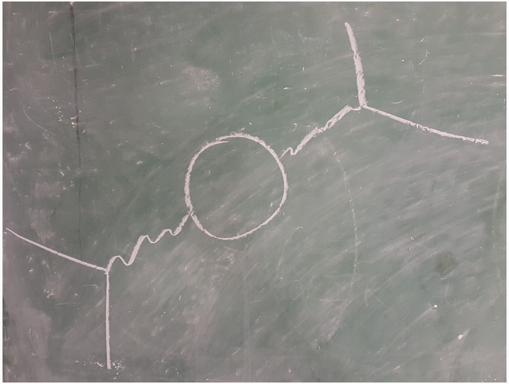
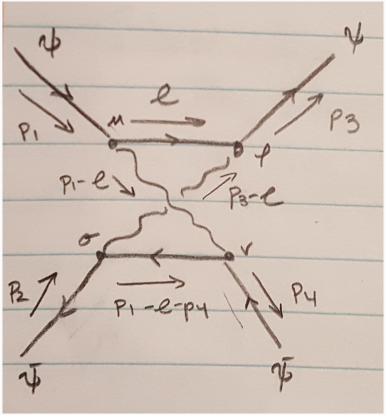
Photon Correction



Box

- There are two diagrams that have an anomalous magnetic moment-type correction
- There is also a crossed box.

Expressions are quite large and need to be tackled algebraically



$$
= -e^4 \epsilon_{\mu\nu\lambda}\epsilon_{\sigma\rho\beta} \int \frac{d^3k}{(2\pi)^3} \frac{(\bar{u}_3\gamma^\rho[\not{l} + M]\gamma^\mu u_1)}{l^2 - M^2}
\times \frac{(\bar{v}_2\gamma^\sigma[\not{p}_1 - \not{l} - \not{p}_4 + M]\gamma^\nu v_4)}{(p_1 - l - p_4)^2 - M^2}
\times \frac{(p_1 - l)^\lambda (p_3 - k)^\beta}{(p_1 - l)^2 (p_3 - l)^2}
$$

On the programming side, made code to

1. Take diagram input and reduce products of gamma functions using

$$\gamma^\alpha \gamma^\beta = g^{\alpha\beta} - i\epsilon^{\alpha\beta\sigma}\gamma_\sigma$$

   It also has to simplify terms, perform contractions, and deal with epsilon tensors!

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4. Collect the expression into sums of terms proportional to known master integrals

OUR RESEARCH: ANALYTIC PROBLEMS

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3. For the box diagrams we cannot solve the integrals with Feynman parameters, we trade them for a (double) Mellin-Barnes integral

$$\frac{1}{(X+Y)^\lambda} = \frac{1}{\Gamma(\lambda)} \frac{1}{2\pi i} \int_{-i\infty}^{+i\infty} dz \Gamma(\lambda + z) \Gamma(-z) \frac{Y^z}{X^{\lambda+z}}$$

4. Completed computation of anomalous magnetic moment and bubble diagram.
5. Checked that these diagrams satisfy the Ward identities imposed by gauge invariance (which had hundreds of terms).

$$\times \quad q^\mu \quad = \quad 0$$

CONCLUSION

SUMMARY:

- Wrote general software to assist scattering calculations to any order, that can deal with epsilons.
- Solved hard integrals.
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WRAP-UP AND FUTURE DIRECTIONS

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FUTURE DIRECTIONS:

- Finish the box integrals
- Combine all the expressions
- Test crossing symmetry