Duality Defects in E_8

for Global Categorical Symmetries (Gong Show)

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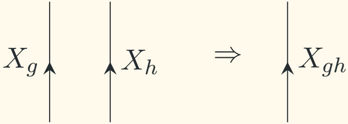
What are Duality Defects?

- **Symmetry defects** for a symmetry group A are modeled by the fusion category Vec_A [Bhardwaj, Tachikawa].



- **Duality defects** are non-invertible topological defect lines which separate a theory from the gauged theory in (1+1)d
  - A duality defect $\mathcal{N}$ enriches this fusion category of symmetry defects with one additional line $\mathcal{N}$

$$X_g \otimes \mathcal{N} = \mathcal{N} = \mathcal{N} \otimes X_g \tag{1}$$

$$\mathcal{N} \otimes \mathcal{N} = \bigoplus_g X_g \tag{2}$$

  - This is a **Tambara-Yamagami category** [Tambara, Yamagami].

- Best example is the **Kramers-Wannier** defect in the Ising.

$$X_1 = \mathrm{Identity}\,, \quad X_\epsilon = \mathbb{Z}_2\ \mathrm{Line}\,, \quad X_\sigma = \mathrm{Duality\ Line}\,, \tag{3}$$

How do we study duality defects?

- Study duality defects via **Verlinde lines** or **fermionization**.
  1. **Verlinde Lines**. X_σ line in the Ising CFT can be understood as one of the 3 simple anyons in the Ising TFT brought to boundary [Fröhlich, Fuchs, Runkel, Schweigert].
  2. **Fermionization**. $\mathbb{Z}_2$ duality defect lines can be understood as GSO projection (bosonization) of chiral fermion parity $(-1)^{F_L}$ line in a fermionic theory [Thorngren]. E.g. Ising CFT, Monster CFT [Lin, Shao].
- Consider the **chiral E_8 theory**.
  - Theory with 8 independent chiral bosons a^i and a vertex operator Γ_α for each α in the E_8 root lattice.
  - Aka chiral $(E_8)_1$ WZW model; E_8 lattice VOA; or boundary edge theory of Kitaev's E_8 phase.
  - **Unique holomorphic** theory with chiral central charge $c_L = 8$.
- Holomorphic $\Rightarrow \mathrm{Rep}(V)$ is trivial $\Rightarrow$ no Verlinde lines.
- Also want to study non-anomalous $\mathbb{Z}_m$ symmetries for $m > 2$.

How do we Classify Duality Defects?

To classify $\mathbb{Z}_m$ duality defects:

1. Pick a non-anomalous $\mathbb{Z}_m$ symmetry. The uncharged operators form a sub-VOA

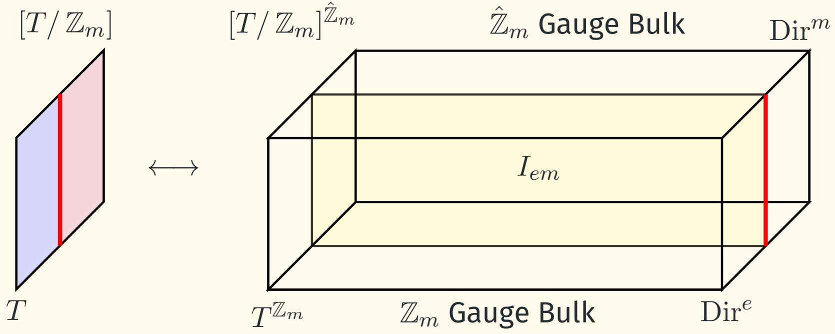
$$V^{\mathbb{Z}_m} \subset V \quad (4)$$

2. $\mathbb{Z}_m$-twisted Hilbert spaces of V decompose over $\mathrm{Irr}(V^{\mathbb{Z}_m})$.

	$[V/\mathbb{Z}_m]$	$[V/\mathbb{Z}_m]^1$	$\cdots$	$[V/\mathbb{Z}_m]^{m-1}$
V	$\mathcal{H}_0^0$	$\mathcal{H}_0^1$	$\cdots$	$\mathcal{H}_0^{m-1}$
V_1	$\mathcal{H}_1^0$	$\mathcal{H}_1^1$	$\cdots$	$\mathcal{H}_1^{m-1}$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
V_{m-1}	$\mathcal{H}_{m-1}^0$	$\mathcal{H}_{m-1}^1$	$\cdots$	$\mathcal{H}_{m-1}^{m-1}$

3. Find all order two automorphisms of $V^{\mathbb{Z}_m}$ which "swap the axes" corresponding to V and its orbifold in the table.

Why does that work?



- In (2+1)d, can couple V to $\mathbb{Z}_m$ gauge theory bulk.
  - V and $[V/\mathbb{Z}_m]$ live on different sides of **electric-magnetic duality interface** [Freed, Teleman], [Gaiotto, JK], [Kaidi's Talk Yesterday!].
  - $\mathcal{N}$ is a twist-line for the $\mathbb{Z}_2$-symmetry of the bulk TFT.
- Can also couple to $\mathcal{Z}(\mathrm{TY}(\mathbb{Z}_m))$-bulk.
  - Pictures are related by gauging $\mathbb{Z}_2^{em}$ symmetry.

What Did We Do?

Tambara-Yamagami Actions on Holomorphic VOAs

Let V be a holomorphic VOA. The inclusion $(V^{\mathbb{Z}_m})^{\mathbb{Z}_2} \subset V$ corresponds to a $\mathbb{Z}_m$ Tambara-Yamagami action on V when the $\mathbb{Z}_2$ action swaps the axes of the metric Abelian group $\mathrm{Irr}(V^{\mathbb{Z}_m})$.

$$Z_V[0, \mathcal{N}_{\hat{\sigma}}] = \mathrm{tr}_{V^G}\, \hat{\sigma} q^{L_0 - \frac{c}{24}} \,. \tag{5}$$

- Compute all $\mathbb{Z}_2$, $\mathbb{Z}_3$, $\mathbb{Z}_4$, and $\mathbb{Z}_5$ TY actions on E_8 theory.
  - Compute their twisted partition functions.
  - Check $\mathbb{Z}_2$ results against answer from fermionization.
  - Check $\mathbb{Z}_3$ defect against conformal inclusion with Potts CFT.
- Additional associator data for the Tambara-Yamgami category. **A lot of physical data!**
- Opportunities to study triality defects since $\mathfrak{so}_8 \subseteq \mathfrak{e}_8$.

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