Holomorphic QFTs: Higher Structures and Bootstrap

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Holomorphic QFTs and Holomorphic Twists

- **Holomorphic QFTs** depend only on the complex structure of the underlying manifold [Johansen], [Nekrasov], [Costello], [Williams], ...
  - Local operators carry structure of a **Holomorphic Factorization Algebra** [Costello, Gwilliam].
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  - Anti-holomorphic translations are Q-exact, so twisted theory is (cohomologically) holomorphic

$$\{Q, \bar{Q}_{\dot{\alpha}}\} = \partial_{\bar{z}^{\dot{\alpha}}} \tag{1}$$

  - Infinite dimensional symmetry enhancements analogous to Virasoro and Kac-Moody [Gwilliam, Williams].
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- Operators captured by holomorphic twist are those counted by **superconformal index** [Saberi, Williams]

$$\mathcal{I} = \mathrm{Tr}(-1)^F p^{j_1+j_2-r/2} q^{j_1-j_2-r/2} e^{-\beta\{Q_-,S^-\}} \tag{2}$$

Higher Brackets and Homotopy Transfer

- Theories are equipped with local product called **λ-Bracket**

$$\{\mathcal{O}_1, \mathcal{O}_2\}_\lambda = \oint_{S^3} e^{\lambda \cdot z} d^2 z \; \mathcal{O}_1(z, \bar{z})\, \mathcal{O}_2(0) \tag{3}$$

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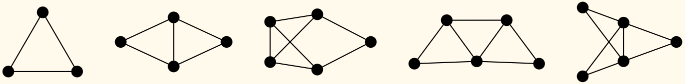
- **All perturbative corrections** are contained in the higher brackets of the free holomorphic factorization algebra!

$$\mathbf{Q}\,\mathcal{O} = \{\mathcal{I}, \mathcal{O}\}_0 + \{\mathcal{I}, \mathcal{I}, \mathcal{O}\}_{0,0} + \{\mathcal{I}, \mathcal{I}, \mathcal{I}, \mathcal{O}\}_{0,0,0} + \ldots . \tag{6}$$

  - [Tree-level] $\sim 1\,\mathcal{I}$, [1-Loop] $\sim 2\,\mathcal{I}$'s , etc.

Feynman Diagrams and Bootstrap

- Feynman diagrams in theory must be **Laman graphs**.



  - Arbitrary integral takes the form:

$$\mathcal{I}_\Gamma[\lambda; z] \equiv \int_{\mathbb{R}^{4|\Gamma_0|-4}} \bar{\partial} \left[\prod_{e \in \Gamma_1} \mathcal{P}(x_{e_0} - x_{e_1} + z_e, \bar{x}_{e_0} - \bar{x}_{e_1}) \right] \left[\prod_{v \in \Gamma_0'} e^{\lambda_v \cdot x_v} d^2 x_v \right] \quad (7)$$

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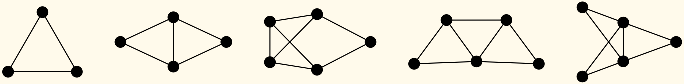

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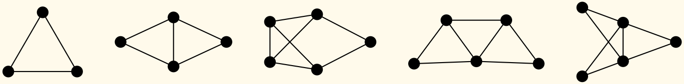

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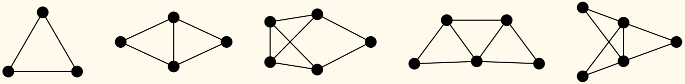

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$$\sum_S \sigma(\Gamma, S)\, \mathcal{I}_{\Gamma[S]}[\lambda + \partial_{z'}; z]\, \mathcal{I}_{\Gamma(S)}[\lambda'; z'] = 0\,. \tag{8}$$

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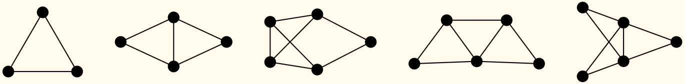

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- Find that quadratic identities are sufficient to **bootstrap Feynman integrals** to at least 3-loops (perhaps further!)

Fin