Confinement and Holomorphic Twists of $\mathcal{N} = 1$ SYM

Simons Confinement Collaboration
Inaugural Workshop

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Why Supersymmetry?

- **SUSY QFTs** provide a rich, but tractable, collection of theories in which quantities are exactly computable or highly constrained; including **phases of gauge theories**
  - Non-renormalization theorems [Sohnius, West], [Mandelstam], [Grisaru, Rocek, Siegel], [Seiberg], [Argyres, Plesser, Seiberg] ++
  - Electric-magnetic duality [Montonen, Olive], Seiberg-Witten Theory [Seiberg, Witten], Seiberg duality [Seiberg]
- Insights from SUSY QFTs can tell us about the real world
  - Directly or by studying SUSY breaking [Dine, Yu] ++
  - Tells us which properties of $N_f = N_c = 3$ are generic

Ex. **Super QCD**. $\mathcal{N} = 1\ SU(N_c)$ SYM with N_f flavors exhibits: theories with **confinement** and **chiral symmetry breaking**; confinement and no chiral symmetry breaking; strongly coupled IR CFTs... [Seiberg]

Why Twist?

- The tractability of SUSY QFTs comes, in part, from the existence of various **protected quantities**
  - Can be invariant under deformation of coupling constant
  - Computable in different duality frames; probe NP physics
  - Ex. F-terms and chiral operators. Protected by two of the four supercharges. Constrain the space of supersymmetric vacua and deformations of the theories [Seiberg].
  - Ex. Superconformal index counts local operators with signs [Kinney, Maldacena, Minwalla, Suvrat], [Romelsberger], [Dolan, Osborn]
- **Twisting** isolates these protected quantities [Witten]
  - The twisted theory is completely **mathematically rigorous**.

Goal

Explain some properties of holomorphic twists of SUSY QFTs, and the **holomorphic confinement** of $\mathcal{N} = 1$ SYM.

- **Supersymmetry** enhances Poincaré symmetry

$$\{Q^A_\alpha, \bar{Q}_{\dot{\beta}B}\} = \delta^A_B P_{\alpha\dot{\beta}}\,, \tag{1}$$

$$\{Q^A_\alpha, Q^B_\beta\} = \{\bar{Q}_{\dot{\alpha}A}, \bar{Q}_{\dot{\beta}B}\} = 0\,. \tag{2}$$

- Given any SQFT, we obtain the **Holomorphic Twist** by taking cohomology of any one nilpotent supercharge, e.g. $Q := Q_-$

$$Q^2 = 0\,, \quad Q\text{-Closed: } [Q, \mathcal{O}] = 0\,, \quad Q\text{-Exact: } [Q, \Lambda]\,, \tag{3}$$

  - Anti-holomorphic translations are Q-exact, so twisted theory is (cohomologically) **holomorphic** [Johansen], [Nekrasov], [Costello]

$$\{Q, \bar{Q}_{\dot{\alpha}}\} = \partial_{\bar{z}^{\dot{\alpha}}} \tag{4}$$

  - Most available & least forgetful twist: only needs $\mathcal{N} = 1$ SUSY.

- Operators captured by holomorphic twist are those counted by the **superconformal index** [Saberi, Williams]

$$\mathcal{I} = \mathrm{Tr}(-1)^F p^{j_1+j_2-r/2} q^{j_1-j_2-r/2} e^{-\beta\{Q_-, S_+\}} \tag{5}$$

HOLOMORPHIC QFTS

- **Holomorphic QFTs** depend only on the complex structure of the underlying manifold [Johansen], [Nekrasov], [Costello], [Williams], ++ .
  - Local operators carry structure of a **Holomorphic Factorization Algebra** [Costello, Gwilliam].
  - Example. 2d Chiral Algebra and/or Vertex Algebra
- Infinite dimensional symmetry enhancements analogous to Virasoro and Kac-Moody [Gwilliam, Williams].
- Theories are equipped with local product called **λ-Bracket**

$$\{\mathcal{O}_1, \mathcal{O}_2\}_\lambda = \oint_{S^3} e^{\lambda \cdot z}\, \mathcal{O}_1(z, \bar{z})\, \mathcal{O}_2(0)\, d^2 z \tag{6}$$

  - **Higher brackets** describe homotopy between lower brackets

$$\{\mathcal{O}_1, \mathcal{O}_2, \ldots, \mathcal{O}_{n+1}\}_{\lambda_1, \ldots, \lambda_n} \tag{7}$$

- Stress: Statement about the OPE of a subsector of the **original physical theory**, *not* a deformed/modified theory.

Adding Interactions

- Polynomials in fields and derivatives $\rightsquigarrow$ **Free Cohomology** $\mathcal{V}$
  - **Interacting quantum theory** is obtained from underlying free-classical theory $\mathcal{V}$ as cohomology of a new operator

$$\mathbf{Q} = Q_0 + Q_1 + Q_2 \dots . \tag{8}$$

    where Q_n is computed by n-loop Feynman diagrams.

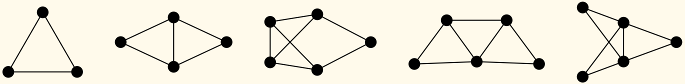
- In interacting quantum theory **all perturbative corrections** are contained in the higher brackets of the free holomorphic factorization algebra! [Budzik, Gaiotto, JK, Williams, Wu, Yu]

$$\mathbf{Q}\,\mathcal{O} = \{\mathcal{I}, \mathcal{O}\}_0 + \{\mathcal{I}, \mathcal{I}, \mathcal{O}\}_{0,0} + \{\mathcal{I}, \mathcal{I}, \mathcal{I}, \mathcal{O}\}_{0,0,0} + \dots . \tag{9}$$

  - [Tree-level] $\sim$ 1 $\mathcal{I}$, [1-Loop] $\sim$ 2 $\mathcal{I}$'s , etc.

Feynman Diagrams

- Feynman diagrams are **Laman graphs**. [Budzik, Gaiotto, JK, Wu, Yu]



  - Arbitrary holomorphic-topological twists in arbitrary dimensions are captured by generalized-Laman graphs [Kontsevich], [Gaiotto, Moore, Witten]

- Arbitrary integral takes the form:

$$\mathcal{I}_\Gamma[\lambda; z] \equiv \int_{\mathbb{R}^{4|\Gamma_0|-4}} \bar{\partial} \left[\prod_{e \in \Gamma_1} \mathcal{P}(x_{e_0} - x_{e_1} + z_e, \bar{x}_{e_0} - \bar{x}_{e_1}) \right] \left[\prod_{v \in \Gamma_0'} e^{\lambda_v \cdot x_v} d^2 x_v \right] \tag{10}$$

  - Change of variables maps integral to Fourier transform of a polytope in loop momenta, the **operatope**.
  - Feynman integrals satisfy infinite collection of geometric **quadratic identities**; enforcing associativity
  - Can **bootstrap** all Feynman integrals from these identities.

Twisting $\mathcal{N} = 1$ SYM

- $\mathcal{N} = 1$ **SYM** is $SU(N)$ gauge theory with an adjoint fermion

$$\mathcal{L} = \int d^2\theta \frac{-i}{8\pi} \tau \, \mathrm{tr} \, W_\alpha W^\alpha + \mathrm{c.c.} \tag{11}$$

- Twist is identified (in a non-trivial way) with a **holomorphic** bc **system** [Costello], [Elliot, Safronov, Williams], [Saberi, Williams]
  - Fields are collected in adjoint bosonic superfield b and (co)adjoint fermionic superfield c.

- The Lagrangian of this theory is

$$\mathcal{L}_{\mathrm{twisted}} = \mathrm{Tr} \, b \left(\bar{\partial} c - \frac{1}{2}[c, c] \right) + \tau \, \mathrm{Tr} \, \partial_\alpha c \, \partial^\alpha c \, . \tag{12}$$

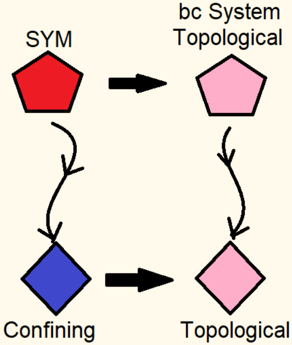
  - Free cohomology

$$\mathbb{C}[b, \partial_\alpha b, \partial_\alpha \partial_\beta b, \ldots, \partial_\alpha c, \partial_\alpha \partial_\beta c, \ldots]^G \tag{13}$$

  - Derivative of the stress tensor is $\partial_\alpha S^\alpha = \partial_\alpha b_A \partial^\alpha c^A$.

Holomorphic Confinement

- Adding **one loop corrections**, we find $\partial_\alpha S^\alpha = Q \operatorname{Tr} b^2$
  - $\partial_\alpha S^\alpha$ generates (among other things) all the remaining spacetime symmetries.
  - Exactness of $\partial_\alpha S^\alpha$ means local operators are invariant under remaining spacetime transformations.
  - Theory becomes **topological** at one loop!



- Being topological is compatible with **confinement**: if topological in the UV, then topological in the IR.
  - Constrains IR physics: the holomorphic twist of the IR must also be topological.
  - We call this **Holomorphic Confinement** [Budzik, Gaiotto, JK, Williams, Wu, Yu]
- **Large N** calculations with **twisted holography** dual.

Conclusion and Future Directions

Conclusion

SUSY QFTs are toy models for real world physics. Tractibility comes from protected quantities; isolated by the twist. These sectors have:

1. Infinite dimensional symmetry enhancements
2. Feynman diagrams which are completely bootstrappable
3. **A novel UV manifestation of confinement**

Future goals:

- Super QCD. Twist has 4d $PSU(N_f|N_f)$ Kac-Moody symmetry.
- **Seiberg Duality**. Requires **non-perturbative corrections**.
- $\mathcal{N} = 4$ SYM. Compute cohomology of $\frac{1}{16}$-BPS states.

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