Decapods : Defects in Quasicrystals

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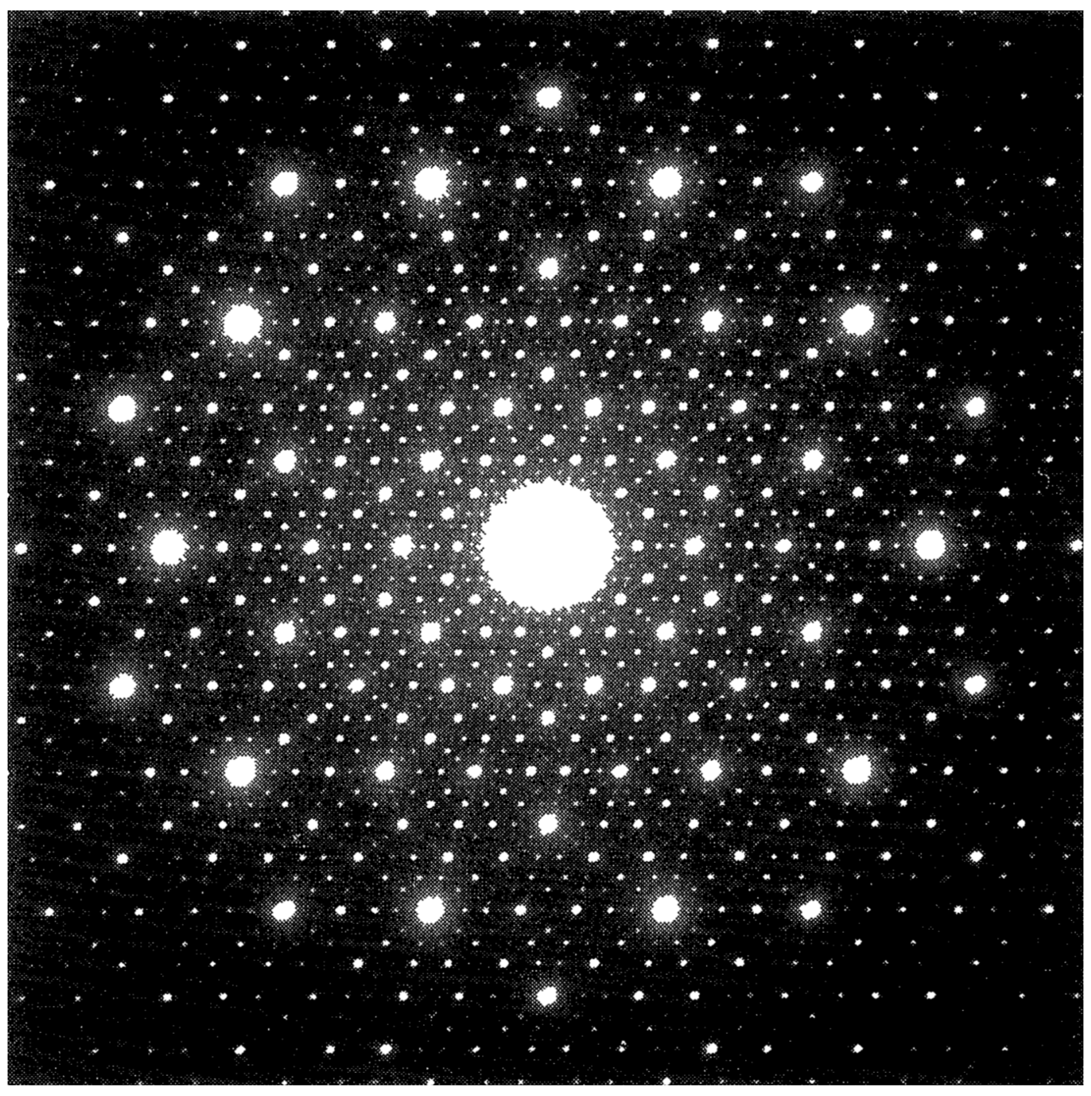
Quasicrystals

Quasicrystals are defined by their diffraction patterns: reciprocal lattice is spanned by $\mathbb{Z}$-linear combinations of $>d$ basis vectors in d dimensions [LS].

- In real space: finite translations and discrete rotations *almost* overlap.
- In momentum space: can possess **classically forbidden symmetries**.

Ex. Diffraction Pattern of Al-Ni-Co [Hir]

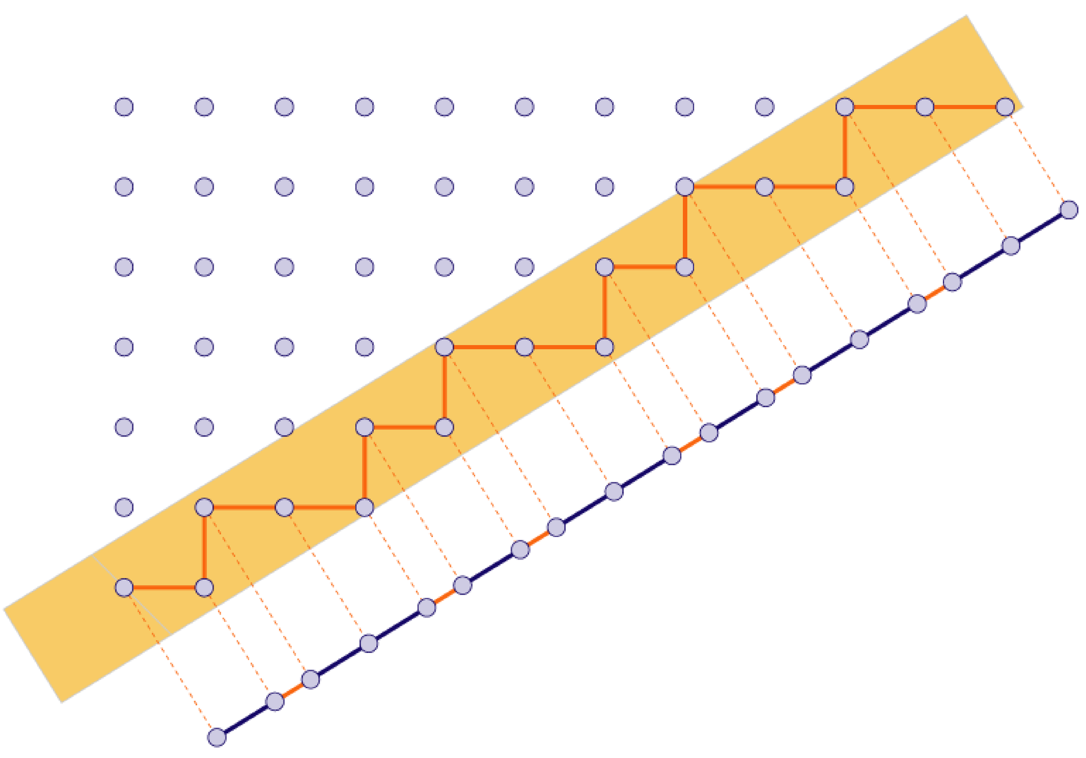
- Classically forbidden 10-fold rotational symmetry.
- Below, the Bragg peaks should densely fill the plane. The diffraction pattern is not completely white because any real diffraction experiment has an IR cutoff.



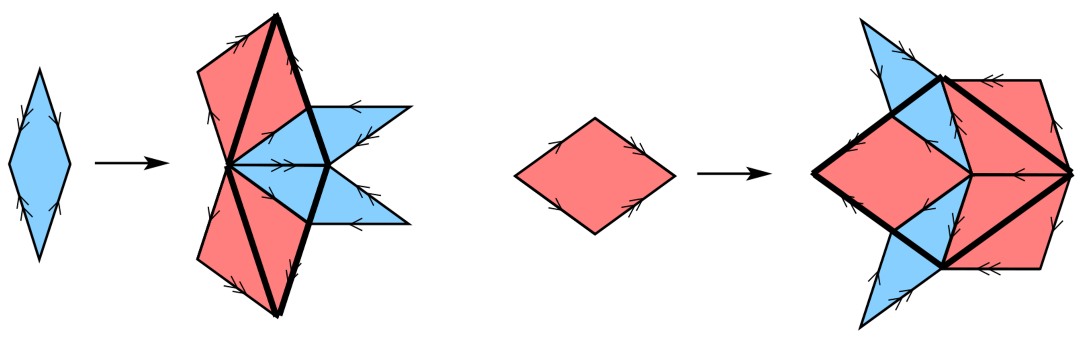
How do I Make a Quasicrystal?

There are many ways to make a quasicrystal:

- **Cut and Project**. Take any regular lattice in $2d$ dimensions. Cut the lattice with a d dimensional plane at an **irrational angle**. Project all sites within an acceptance band onto a d-dimensional surface.



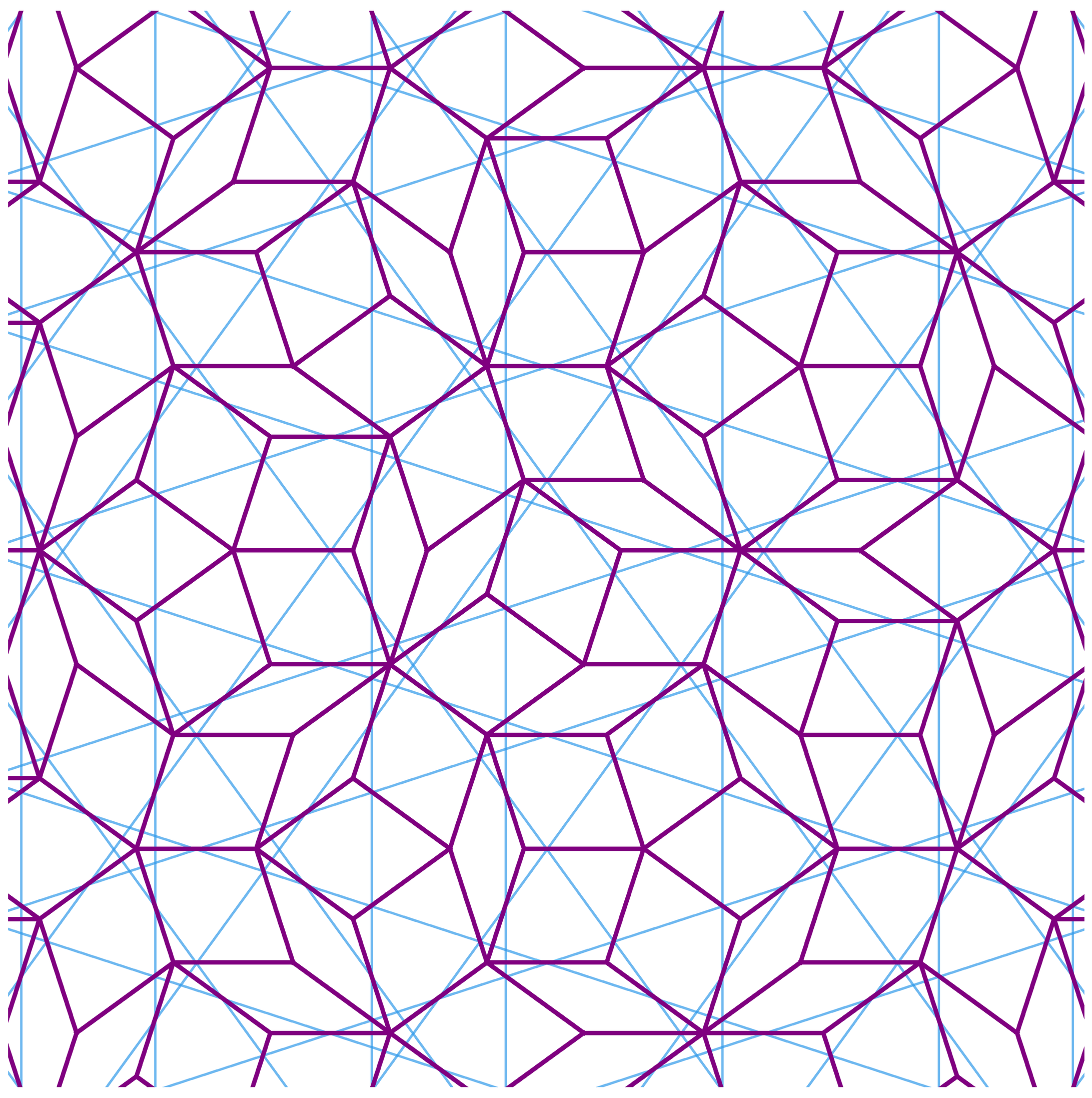
- **Local Matching Rules**. Mark tiles with arrows or colours, and match the patterns or colours across tile edges (see below).



- **Substitution (or Inflation/Deflation) Rules**. Start with a finite seed tiling and decimate each tile into a collection of tiles in a prescribed way (see above). Scale up the tiling. Repeat ad infinitum.

More: Ammann lines and de Bruijn's grid method.

Penrose Tilings



Penrose tilings are the most well-known quasicrystals, with points of 10-fold symmetry [Gard].

- There are an **uncountable-∞** of Penrose tilings.
- Penrose tilings are **Locally Indistinguishable**: any finite sized patch of tiles in one Penrose tiling can be found in any other.

Ammann Lines and LRO

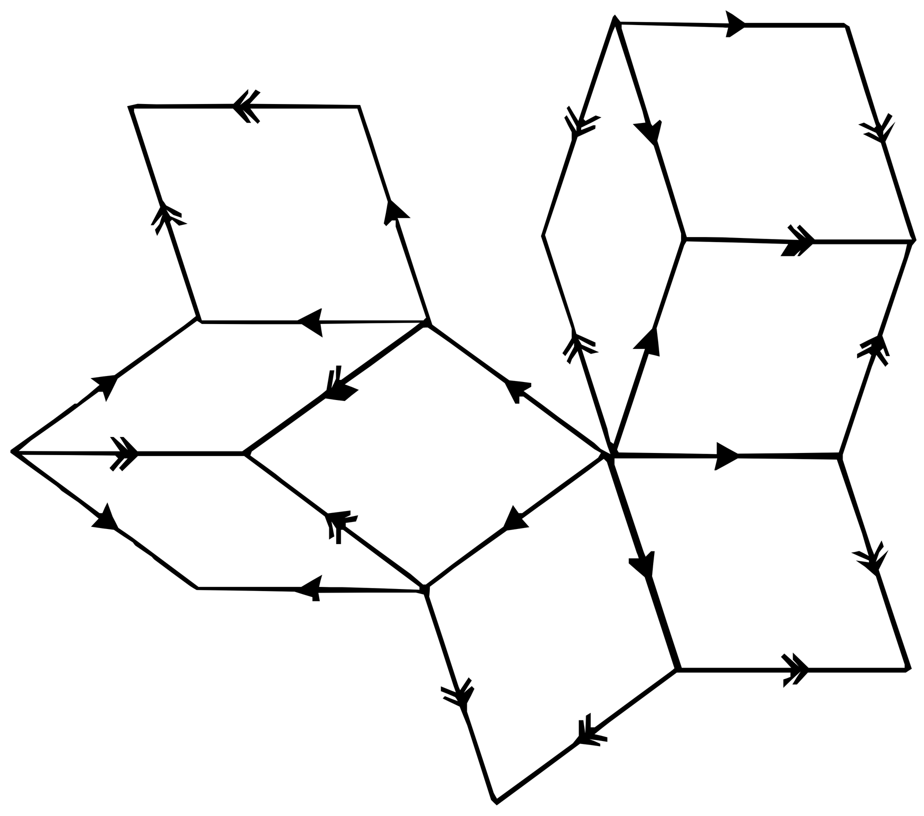
Ammann lines (marked above) are one way to construct a Penrose tiling: lines must be unbroken.

- Equivalent to local matching rules.
- Ammann lines show **long-range order** in quasicrystal: placing one tile forces a whole line of options along each Ammann line.
- Ammann lines form 5 intersecting 1d Fibonacci quasicrystals $\Rightarrow$ non-periodicity of Penrose tiles.

Defects from Local Rules

Placing tiles following local matching rules, it is possible to create a patch which cannot be extended to cover all of $\mathbb{R}^2$.

- Highlights non-locality of quasicrystals again.
- No tile can fill the hole below.



Further References

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[LS] D. Levine, P.J. Steinhardt, *Quasicrystals. I. Definition and structure*, Phys. Rev. B, Vol. 34-2, 1986.
[Jeong] H-C. Jeong, *Growing Perfect Decagonal Quasicrystals by Local Rules*, Phys. Rev. Lett., Vol. 98-13, 2007.
[Gard] M. Gardner, *Penrose Tiles to Trapdoor Ciphers*, CUP, 1997.
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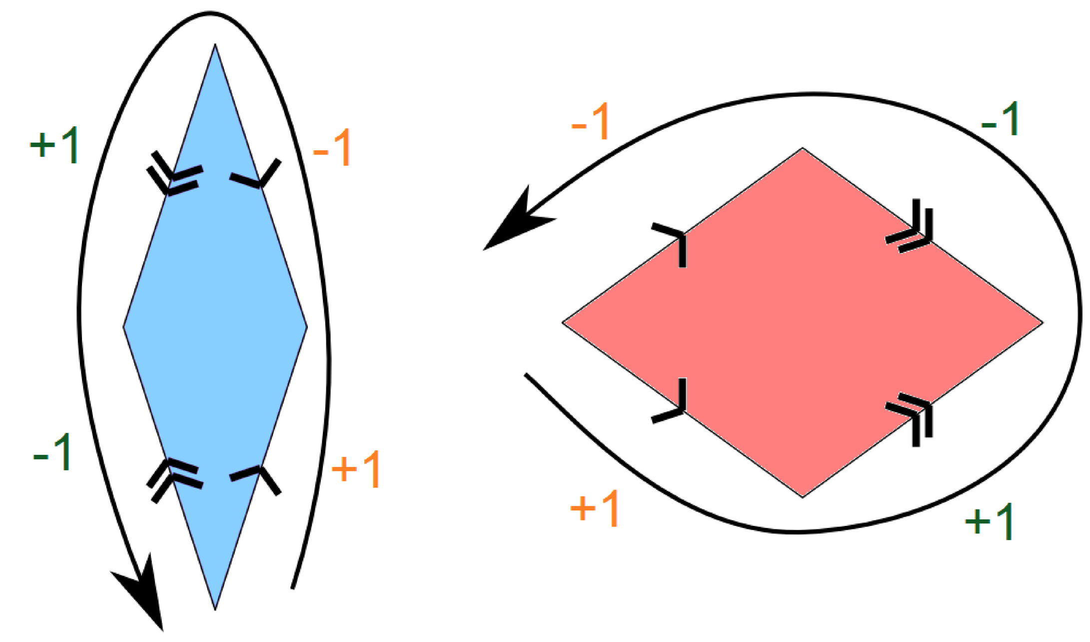
Charges in the Penrose Tiling

Treating the matching rules for the Penrose rhombs as **charges**, we see that an individual Penrose tile has no net charge.

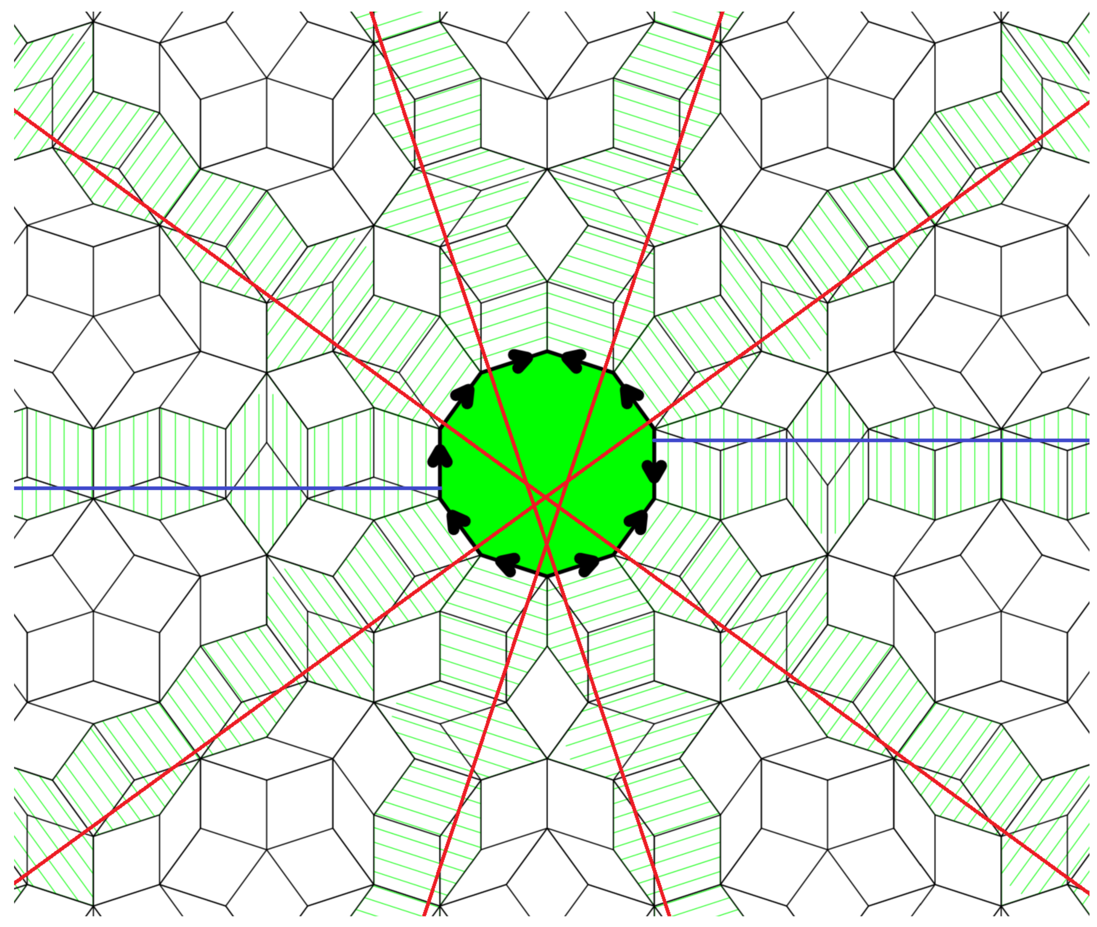
- Every simply connected patch of tiles has 0 net single-arrow charge and double-arrow charge.



Conway's Decapods

Conway's Decapods are defected Penrose tilings

- Central decagonal hole with infinitely long **worms** (shaded green) radiating outwards.
- Any of the worms can be flipped to produce another decapod defect. 2^{10} decapods.
  - 62 decapods modulo rotations and flips.
  - 61 of 62 are genuine defects (cannot be filled in).
- Defect decapods can be identified from Ammann lines which do not match (blue) across the hole.



The decapods are not simply connected, and so may **carry non-trivial charge**

- Can accumulate non-zero net single-arrow charge

Charge 10 $\sim$ 1 Decapod	Charge 4 $\sim$ 12 Decapods
Charge 8 $\sim$ 1 Decapod	Charge 2 $\sim$ 22 Decapods
Charge 6 $\sim$ 5 Decapods	Charge 0 $\sim$ 21 Decapods

Conway's Defect Conjecture

Every possible hole is equivalent to a decapod hole by re-arranging tiles around the hole; taking away or adding a finite number of pieces [Gard].

Understanding Conway's Conjecture

Decapods are evocative of other phenomena:

- Decapods have semi-infinite Ammann lines which "miss" each other; **pinning** them.
  - Analogous to **fractons**?
- Binary charge is not strong enough to distinguish decapods. Is there a **non-abelian charge** that can be assigned that will lift their degeneracy?
- No continuum description of decapods.