Twisted Tools for (Untwisted) Quantum Field Theory

Yale Geometry, Symmetry and Physics Seminar

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Outlines and Punchlines 1/2

- Discuss ideas from formal deformation theory in QFT
  - Familiar example is Ocneanu rigidity of fusion categories
- QFTs have "higher" **multilinear k-ary operations** ("brackets")

$$\{-,-,\ldots,-\} \tag{1}$$

  - Control: **deformations**, (generalized) **OPEs**, and **anomalies**
  - ∞-algebras, factorization algebras, and operads
- Familiar to high energy physicists and mathematicians who have studied twisted SQFTs (descent relations)
  - Not limited to twisted scenarios
- Can go very far in the case of (mixed) Holomorphic and/or Topological (HT) theories

Outlines and Punchlines 2/2

I. Introduce the eta-function and higher brackets
II. Computing the eta-function and generalizations
III. Holomorphic-Topological diagrams and integrals
IV. A non-renormalization theorem for HT theories
V. Applications to SQFTs

Three Takeaways

1. η-vector exists and contains anomalies/OPEs/more
2. η-vector is computable, especially in HT scenarios
3. Non-renormalization theorem for HT theories

The η-Function

Deformations of QFTs

- Given a QFT T, it can be deformed by turning on interactions

$$S_T + \sum_i g^i \int_{\mathbb{R}^d} \mathcal{O}_i(x) d^d x \qquad (2)$$

  - g^i are coordinates on theory space
  - Work perturbatively in couplings g^i
- Defines a **formal pointed neighbourhood** $\mathcal{D}[T]$ of T, consisting of all effective QFTs obtained by perturbative deformation of T
  - Pointed because there is a distinguished point, called T.
  - Formal because we only consider deformations in an infinitesimal nbd of T (we are not at finite coupling).
  - Think of formal/infinitesimal as synonym for “perturbative”

The Beta Function

- Generic QFT (point) is not scale invariant
  - Scale transformation on T is traded for a change of the couplings e.g. integrating out DoF modifies couplings
- We encode infinitesimal scale transformations in vector field on theory space, the **beta function**

$$\boldsymbol{\beta} = \sum_i \beta^i(g) \frac{\partial}{\partial g^i} \tag{3}$$

  - Perturb around (typically free) scale-invariant theory, $\boldsymbol{\beta} = 0$
  - Deformations of T preserving scale invariance are zeroes of $\boldsymbol{\beta}$
- The coefficients $\beta^i(g)$ are power series in g

$$\beta^i(g) = \underbrace{(d - \Delta_i)}_{\text{Classical}} g^i + O(g^2) \tag{4}$$

  - Tune relevant terms to 0 and study $\boldsymbol{\beta}$ as a measure of scale generated by "quantum effects"

- Quantization of non-abelian gauge theories is hard: formulated redundantly in exchange for other properties

$$Z = \int [DA D\bar{\psi} D\psi] e^{-S[A,\bar{\psi},\psi]} =: \int [D\Phi] e^{-S[\Phi]} \,, \tag{5}$$

  - Introduce a gauge fixing procedure and Fadeev-Poppov ghosts b and c

$$Z = \int [D\Phi DB_A db_A dc^\alpha] e^{-S[\Phi]+iB_A F^A[\Phi]-b_A c^\alpha \delta_\alpha F^A[\Phi]} \tag{6}$$

- Gauge fixed action still has residual nilpotent odd global symmetry involving fields and ghosts, called **BRST symmetry**.

$$\begin{aligned} \delta_{\mathrm{BRST}} \Phi &= -i\epsilon c^\alpha \delta_\alpha \Phi \,, & \delta_{\mathrm{BRST}} B_A &= 0 \,, \\ \delta_{\mathrm{BRST}} c^\alpha &= \frac{i}{2} \epsilon f^\alpha_{\beta\gamma} c^\beta c^\gamma \,, & \delta_{\mathrm{BRST}} b_A &= \epsilon B_A \,, \end{aligned} \tag{7}$$

  - Physical theory can be identified with Q_{BRST}-cohomology

The BV-BRST Formalism 2/2

- Focus on theories defined in BV-BRST formalism:
  - T is embedded in a bigger ambient theory $\widetilde{T}$ with ghosts, anti-ghosts, anti-fields, etc.
  - Grassmann odd nilpotent symmetry Q_{BRST}
  - Observables in T are recovered from $\widetilde{T}$ by taking Q_{BRST} coho

$$\begin{aligned} \text{Ops}_T &= (\text{Ops}_{\widetilde{T}}, Q_{\text{BRST}}) \\ \text{Int}_T &= (\text{Ops}_{\widetilde{T}}[dx], \mathrm{d} + Q_{\text{BRST}}) \end{aligned} \tag{8}$$

- i.e. we will work in BV formalism
  - Essential to quantizing p-form gauge theories, theories which only close on-shell, field-dependent structure constants, or theories with other complicated constraints
  - Not restricted to such complicated theories either

The Eta Function

- Can compute analog of β **for any type of transformation**.
  - Ex. Non-relativistic scale transformations $(t, x) \mapsto (\lambda^z t, \lambda x)$
  - Ex. Anomalous axial transformation on θ angle in gauge theory
- Consider $T \hookrightarrow (\widetilde{T}, Q_{\mathrm{BRST}})$ described in a BRST formalism in terms of ambient $\widetilde{T}$
  - To deform T, we deform $\widetilde{T}$ without breaking BRST symmetry
  - Consider deformations of $\tilde{T}$ with Grassmann odd couplings, non-trivial ghost number, etc. This is a formal pointed dg-supermanifold $\mathcal{D}[\widetilde{T}]$.
- BRST symmetry will be encoded in a vector field

$$\boldsymbol{\eta} = \sum_i \eta^i(g) \frac{\partial}{\partial g^i} \tag{9}$$

  - Linear term tells us if adding an interaction $\mathcal{I}$ explicitly/classically violates BRST symmetry
  - Higher order terms do so "quantum mechanically"

- Since $Q^2 = 0$, the eta function $\boldsymbol{\eta}^2 = 0$.
  - Wess-Zumino consistency condition for BRST symmetry
  - Gives **quadratic constraints** on coefficient functions $\eta^i(g)$

$$\eta^i(g) = \sum_{n>0} \frac{1}{n!} \sum_{j_1 \cdots j_n} \eta^i_{j_1 \cdots j_n} g^{j_1} \cdots g^{j_n} \tag{10}$$

- Define the following multilinear operation $\mathrm{Int}^{\otimes n} \to \mathrm{Int}$

$$\{g^{j_1}\mathcal{I}_{j_1}, \cdots, g^{j_n}\mathcal{I}_{j_n}\} = \eta^i_{j_1 \cdots j_n} g^{j_1} \cdots g^{j_n} \mathcal{I}_i \tag{11}$$

  - The BRST variation becomes

$$\boldsymbol{\eta}\mathcal{I} = \{\mathcal{I}\} + \frac{1}{2!}\{\mathcal{I},\mathcal{I}\} + \frac{1}{3!}\{\mathcal{I},\mathcal{I},\mathcal{I}\} + \ldots \tag{12}$$

$\boldsymbol{\eta}^2 = 0 \quad \Leftrightarrow$ The coefficients $\eta^i_{j_1 \cdots j_n}$ and brackets $\{\cdot, \ldots, \cdot\}$ define an $L_\infty[1]$**-algebra** structure on In.

η-Function Calculations and Data

Basic Eta-Function Calculation - General 1/2

- η is not just abstract fun, it is computable fun
- Correlation functions of T deformed by $\mathcal{I}$ are correlation functions of T with additional insertions

$$\langle \mathcal{O}_1(x_1) \cdots \mathcal{O}_n(x_n) \rangle_{T+\mathcal{I}} = \left\langle \mathcal{O}_1(x_1) \cdots \mathcal{O}_n(x_n) e^{g \int d^d x \, \mathcal{I}} \right\rangle_T \quad (13)$$

  - At **linear order**, the BRST anomaly generated by $\mathcal{I}_i$ is just

$$\int_{\mathbb{R}^d} [Q, \mathcal{I}_i] \, . \quad (14)$$

  - Write

$$[Q, \mathcal{I}_i] = \sum_j \eta_i^j \mathcal{I}_j + d\mathcal{J}_i \quad (15)$$

- In perturbation theory, higher order terms

$$O(g^n) \sim \int_{\mathbb{R}^{dn}} \mathcal{I}(y_1) \cdots \mathcal{I}(y_n) \quad (16)$$

  - Need **regularization** to avoid UV divergences from colliding $\mathcal{I}$

Basic Eta-Function Calculation - General 2/2

- E.g. **at $O(g^2)$** we can regularize the deformation to

$$\int_{\mathbb{R}^{2d}} f_\epsilon^{(2)}(x_1, x_2)\mathcal{I}(x_1)\mathcal{I}(x_2) \tag{17}$$

- Now we can compute

$$\left[Q, \int_{\mathbb{R}^{2d}} f_\epsilon^{(2)}(x_1, x_2)\ \mathcal{I}(x_1)\mathcal{I}(x_2)\right]\Big|_{d\mathcal{J}} \tag{18}$$

$$= -\int_{\mathbb{R}^{2d}} df_\epsilon^{(2)}(x_1, x_2)(\mathcal{I}(x_1)\mathcal{J}(x_2) + \mathcal{J}(x_1)\mathcal{I}(x_2)) \tag{19}$$

- With a sharp cutoff (**point-splitting**) this becomes

$$\{\mathcal{I}, \mathcal{I}\}(x_2) \overset{\text{Sharp Cutoff}}{=} \int_{|x_{12}|=\epsilon} \mathcal{I}(x_1)\mathcal{J}(x_2) + \mathcal{J}(x_1)\mathcal{I}(x_2) \tag{20}$$

  - Anomaly appears in point-splitting regularization because total derivative terms give a boundary contribution.

Basic Eta-Function Calculation - Concrete 1/2

- 2d S_{Matter} with G global symmetry and G gauge theory

$$S_T = -\frac{1}{4}\int d^2x\, F_{\mu\nu}F^{\mu\nu} + S_{\text{Matter}} \tag{21}$$

Ex. Free fermions with vector current $J^\mu_a = \bar{\psi}\gamma^\mu t_a \psi$.

  - Study the **interaction** $\mathcal{I} = A_\mu J^\mu$

- Add ghost and auxiliary fields $T \hookrightarrow (\widetilde{T}, Q)$
  - BRST transformation of $\mathcal{I}$ gives:

$$\delta_{\text{BRST}}(A_\mu J^\mu) = (\epsilon D_\mu c)J^\mu + A_\mu(i\epsilon g c J^\mu) = \epsilon\partial_\mu c J^\mu\,, \tag{22}$$

  - See $\mathcal{I}$ is BRST-closed up to total derivative $\mathcal{J} = cJ$
  - Term can potentially cause BRST anomaly

Basic Eta-Function Calculation - Concrete 2/2

- The two-bracket receives a contribution from the 2d JJ OPE:

$$\{\mathcal{I},\mathcal{I}\}(x_2) = \int_{|x_{12}|=\epsilon} :AJ:(x_1)\ :cJ:(x_2) + :cJ:(x_1)\ :AJ:(x_2)$$
$$= \oint_{S^1_{x_2}} \left(:A(x_1)c(x_2): + :c(x_1)A(x_2): \right) \langle J(x_1)J(x_2)\rangle$$
$$= \#\ :c\, dA:\ (x_2)\,. \qquad (23)$$

  - We use $JJ \sim |x_{12}|^{-2}$, taylor expanded, and integrated by parts
  - $\#$ denotes combinatorial and rep-theoretic factors

- Recover well-known **1-loop anomaly for G-gauge theory**

$$\{A_\mu J^\mu, A_\nu J^\nu\} = \#\, cF_{12}\,. \qquad (24)$$

  - In $2k$-dim, you recover anomaly from $(k+1)$-ary bracket

Perturbative Corrections to Q

- Callan-Symanzik equation says renormalized correlators are independent of (arbitrary) renormalization scale μ:

$$\mu \frac{d}{d\mu} G^{(n)} = \left(\mu \frac{\partial}{\partial \mu} + \boldsymbol{\beta} + \boldsymbol{\gamma} \right) G^{(n)} = 0 \tag{25}$$

- For BRST symmetry, we have $\mathcal{L}_{\boldsymbol{\eta}}$. Nilpotency implies:

$$\mathcal{L}_{\boldsymbol{\eta}}^2 = \{\boldsymbol{\eta}, \mathbf{Q}\} + \mathbf{Q}^2 = 0 \tag{26}$$

  - Coefficients $Q^i(g)$ of $\boldsymbol{Q}$ can be identified as coefficients for multilinear operations $\mathrm{Int}^{\otimes n} \otimes \mathrm{Op} \to \mathrm{Op}$
  - Local operators have a (right) L_∞-module structure

$$\boldsymbol{Q}\, \mathcal{O} = \{\mathcal{O}\} + \{\mathcal{I}, \mathcal{O}\} + \frac{1}{2}\{\mathcal{I}, \mathcal{I}, \mathcal{O}\} + \cdots . \tag{27}$$

- Can systematically compute corrections to “semi-chiral ring” in SUSY Twists

Recap and Further Connections

- Recap:
  - Deformations are integral to our understanding of QFT
  - Working in a BV-BRST formalism, we can introduce η that tracks violation of BRST symmetry due to interactions
  - η defines an L_∞-algebra on interactions and Maurer-Cartan equation is solved by well-defined deformations
  - η contains useful information like anomalies
- Descent operations in twisted theories
  - Higher brackets appear by colliding/integrating descendants
  - E.g. OPE and secondary product of cohomological TFTs is a 2-ary bracket
  - [Bomans, Wu] compute higher-central charges (a and c) of 4d SUSY gauge theories from brackets

Generalizations

- Can consider **position dependent interactions**: causes momentum-inflow p^i at each vertex
  - L_∞-brackets extended to $\otimes_i \mathrm{In}_{p^{(i)}} \to \mathrm{In}_{\sum_i p^{(i)}}$
  - Momentum-coloured operad

$$\{\mathcal{I}_{i_1} \,_{p^{(1)}}\, \mathcal{I}_{i_2} \,_{p^{(2)}}\, \ldots \,_{p^{(n-1)}}\, \mathcal{I}_{i_n}\} \tag{28}$$

- Distinguished subcase: holomorphic theories with holomorphic momentum λ recovers λ-brackets and higher n-Lie or homotopy conformal algebras
- **Auxiliary and defect systems**: the brackets of $T \times T_{\text{probe}}$ extract information about T

  Ex. 't Hooft anomaly of S_{Matter} is apparent in the non-trivial bracket when coupled to G-gauge theory T_{probe}

  Ex. If T is topological QM, brackets of T recover Moyal commutator. Brackets with an auxiliary fermion recovers full Moyal-star product. 1d-topological defect brackets have A_∞.

Holomorphic-Topological Theories

- "**Holomorphic-Topological**" means flat spacetime has structure of $\mathbb{C}^H \times \mathbb{R}^T$ with coords $(x^{\mathbb{C}}, \bar{x}^{\mathbb{C}}, x^{\mathbb{R}})$
  - Anti-holomorphic translations in $\mathbb{C}^H$ and translations in $\mathbb{R}^T$ are gauge symmetries (Q_{BRST}-exact)
  - Interested in theories with action

$$\int_{\mathbb{C}^H \times \mathbb{R}^T} \left[(\Phi, \mathrm{d}\, \Phi) + \mathcal{I}(\Phi) \right] \; d^H x^{\mathbb{C}} \tag{29}$$

  - Φ is a "superfield," and $dx^{\mathbb{R}}$ and $d\bar{x}^{\mathbb{C}}$ are "superspace coordinates" (form-valued fields are superfields).
- Appears in holomorphic-topological twists of SUSY theories.
- In free theory, BRST closed superfields satisfy descent:

$$Q\,\mathcal{O} + \mathrm{d}\,\mathcal{O} = 0 \tag{30}$$

  - Interaction $\mathcal{I}(\Phi)$ is BRST-closed up to total derivative.

Holomorphic-Topological Integrals

- In such theories, we will be interested in brackets of the form

$$\{\mathcal{O}_1 \,{}_{\lambda_1} \cdots {}_{\lambda_{n-1}} \,\mathcal{O}_n\} \tag{31}$$

- The **Feynman integrals** that contribute will take the form:

$$I_\Gamma(\lambda; z) = \int_{\mathcal{M}^{|\Gamma_0|-1}} \left[\prod_{v\in\Gamma_0}^{v\neq v_*} \mathrm{dVol}_v e^{\lambda_v \cdot x_v^{\mathbb{C}}}\right] \mathrm{d}\left[\prod_{e\in\Gamma_1} P_\epsilon(x_{e(0)} - x_{e(1)} + z_e)\right]$$

- Let's count the form degree of the integrand:
  - (Regulated) propagator P_ϵ is an $(H+T-1)$-form
  - $(H+T)\times(|\Gamma_0|-1)$ integration variables: one for each vertex of graph, and throw one vertex away by translation symmetry.
  - $(|\Gamma_1|-1)$ regulated propagators and one $(H+T)$-form cut propagator $\mathrm{d}P_\epsilon(x)$

Holomorphic-Topological Feynman Diagrams

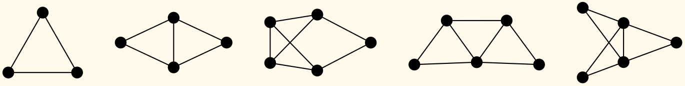
- Non-vanishing Feynman diagrams are **n-Laman graphs**
  - **Global Constraint**

$$n|\Gamma_0| = (n-1)|\Gamma_1| + n + 1 \tag{32}$$

  - **Local Constraint** For subgraphs $\Gamma[S]$

$$n|\Gamma[S]_0| \geq (n-1)|\Gamma[S]_1| + n + 1 \tag{33}$$

  - In particular, $n = H + T$.



- Call a graph "almost n-Laman" of degree $\tau(\Gamma)$ if

$$n|\Gamma_0| = (n-1)|\Gamma_1| + n + 1 + \tau(\Gamma) \tag{34}$$

Quadratic Identities

- $I_\Gamma(\lambda; z)$ has a number of symmetries/identities: symmetries from the graph, and under shifts of z_e.
- Feynman integrals (more generally diagrams) satisfy infinite collections of (geometric) **quadratic identities** associated to each degree-1 almost-Laman graph:

$$\sum_{\text{Laman } S} \sigma(\Gamma, S) I_{\Gamma[S]}(\lambda + \partial; z) \cdot I_{\Gamma(S)}(\lambda; z) = 0 \,. \tag{35}$$

  - Identities imply (higher)-associativity of the accompanying brackets in a diagram-by-diagram way
  - Can **bootstrap** all Feynman integrals from these identities?

A Non-Renormalization Theorem 1/2

- Many interesting scenarios with mixed HT degree.

  Ex. (1+1)d holomorphic boundary of a (2+1)d TFT

  Useful to “forget” some of the structure of the bulk TFT.

- Trade $T = 2$ top. directions for an $H = 1$ holo. direction
  - Topological superfield Φ splits into two fields in the holomorphic theory, a $(0,*)$-form and a $(1,*)$-part:

$$\Phi = \Phi^{(0)} + \Phi^{(1)} dz \, . \tag{36}$$

  - Topological superfield condition also splits

$$(Q + \mathrm{d}_{\mathrm{top}})\Phi = (Q + \mathrm{d}_{\mathrm{Holo}})\Phi + \partial\Phi = 0 \, . \tag{37}$$

- $\partial\Phi$ term is now interpreted as a BRST anomaly due to the holomorphic part of the kinetic term
  - i.e. all top. theory calculations are replaced by calculations in identical holo. theory with extra $(\Phi, \partial\Phi)$ interaction

A Non-Renormalization Theorem 2/2

- Consider a calculation in $(H, T \geq 2)$
  - Convert to an equivalent calculation in $(H+1, T-2)$-theory

$$\begin{array}{ccc} (H, T \geq 2)\text{-theory} & & (H+1, T-2)\text{-theory} \\ \Gamma & \rightsquigarrow & \{\gamma_1, \gamma_2, \dots\} \end{array} \tag{38}$$

  - γ_i will correspond Γ with edges $e_i \in \Gamma_1$ broken into chains of edges $\{f_{i,1}, \dots, f_{i,m}\}$ in all possible ways
  - Each vertex has "two-point interaction" $(\Phi, \partial\Phi)$
  - Each γ_i has $|\Gamma_1| + k$ edges and $|\Gamma_0| + k$ vertices for some $k \geq 0$.
- If γ_i are non-vanishing, they must be $(n-1)$-Laman graphs.
  - Putting the two conditions together, Γ must be a tree for non-vanishing contribution

Non-Renormalization Theorem

All loop graphs in $(H, T \geq 2)$-theories must vanish.

Applications to SQFTs

SUSY and the Semi-Chiral Ring

- **Supersymmetry** enhances Poincaré symmetry

$$\{Q^A_\alpha, \bar{Q}_{\dot\beta B}\} = \delta^A_B P_{\alpha\dot\beta}\,, \tag{39}$$

$$\{Q^A_\alpha, Q^B_\beta\} = \{\bar{Q}_{\dot\alpha A}, \bar{Q}_{\dot\beta B}\} = 0\,. \tag{40}$$

  - In Euclidean signature, $\mathrm{Spin}(4) \cong SU(2)_L \times SU(2)_R$
  - Q_α and $\bar{Q}_{\dot\alpha}$ are two-component spinors

- Pick some supercharge $Q = Q_-$
  - The **semi-chiral ring** of Q consists of all Q-invariant operators

$$[Q, \mathcal{O}] = 0 \tag{41}$$

  - If SUSY isn't broken (so that $Q\,|0\rangle = 0$), then a product of Q-invariant operators satisfies

$$\langle (\mathcal{O} + [Q,\Lambda]) \cdots \rangle = \langle \mathcal{O} \cdots \rangle + \langle [Q, \Lambda \cdots] \rangle = \langle \mathcal{O} \cdots \rangle\ , \tag{42}$$

    So doesn't care about operators modulo $\mathcal{O} \sim \mathcal{O} + [Q,\Lambda]$.

From SUSY to the Holomorphic Twist

- Given any SQFT, we obtain the **Holomorphic Twist** by taking **cohomology** of any one nilpotent supercharge, e.g. $Q := Q_-$

$$Q^2 = 0\,, \quad Q\text{-Closed: } [Q, \mathcal{O}] = 0\,, \quad Q\text{-Exact: } [Q, \Lambda]\,, \tag{43}$$

  - Most available & least forgetful twist: only needs $\mathcal{N} = 1$ SUSY.
  - Cohomology isolates the semi-chiral ring

- Anti-holomorphic translations are Q-exact, so twisted theory is (cohomologically) **holomorphic**

$$\{Q, \bar{Q}_{\dot\alpha}\} = \partial_{\bar{z}^{\dot\alpha}} \tag{44}$$

- Operators captured by holomorphic twist are those counted by the **superconformal index** [Saberi, Williams], [Raghavendran]

$$\mathcal{I} = \mathrm{Tr}(-1)^F p^{j_1+j_2-r/2} q^{j_1-j_2-r/2} e^{-\beta\{Q_-, S_+\}} \tag{45}$$

Twisting $\mathcal{N} = 1$ SYM

- $\mathcal{N} = 1$ **SYM** is $SU(N)$ gauge theory with an adjoint fermion

$$\mathcal{L} = \int d^2\theta \frac{-i}{8\pi} \tau \, \mathrm{tr} \, W_\alpha W^\alpha + \mathrm{c.c.} \tag{46}$$

- Twist is identified (in a non-trivial way) with a **holomorphic** bc **system**
  - Fields are collected in adjoint bosonic superfield b and (co)adjoint fermionic superfield c.
- The Lagrangian of this theory is

$$\mathcal{L}_{\mathrm{twisted}} = \mathrm{Tr} \, b \left(\bar{\partial} c - \frac{1}{2}[c, c] \right) + \tau \, \mathrm{Tr} \, \partial_\alpha c \, \partial^\alpha c \, . \tag{47}$$

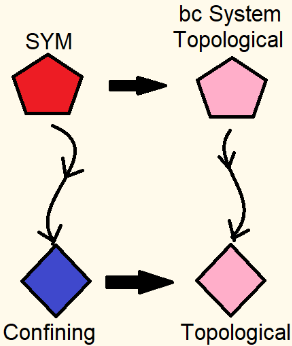
  - Free cohomology

$$\mathbb{C}[b, \partial_\alpha b, \partial_\alpha \partial_\beta b, \ldots, \partial_\alpha c, \partial_\alpha \partial_\beta c, \ldots]^G \tag{48}$$

  - Derivative of the stress tensor is $\partial_\alpha S^\alpha = \partial_\alpha b_A \partial^\alpha c^A$.

Holomorphic Confinement

- $\partial_\alpha S^\alpha$ generates (among other things) all the remaining spacetime symmetries (e.g. holo. translations/rotations).
- Adding **one loop corrections**, we find $\partial_\alpha S^\alpha = Q\,\mathrm{Tr}\,b^2$
  - Exactness of $\partial_\alpha S^\alpha$ means local operators are invariant under remaining spacetime transformations.
  - Theory becomes **topological** at one loop!
- Being topological is compatible with **confinement**: if topological in the UV, then topological in the IR.
  - Constrains IR physics: the holomorphic twist of the IR must also be topological.
  - We call this **Holomorphic Confinement** [Budzik, Gaiotto, JK, Williams, Wu, Yu]



Final Recap

Recap

I. Introduced **η-function**; interactions have L_∞-algebra structure, tracks violation of BRST symmetry by interactions
II. η-function contains familiar data like anomaly data, and briefly discussed the relation to twisted SQFTs
III. Introduced holomorphic-topological theories, and showed brackets are very strongly constrained (**Laman graphs**)
IV. Laman graphs prove associativity relations, and no perturbative corrections when $T \geq 2$ topological directions.
V. Application to Super-Yang Mills and "**holomorphic confinement**"

Three Takeaways

1. η-vector exists and contains anomalies/OPEs/more
2. η-vector is computable, especially in HT scenarios
3. Non-renormalization theorem for HT theories

Fin