Quasicrystals and Decapods

Global Categorical Symmetries, Quantum Field Theory, and Geometry

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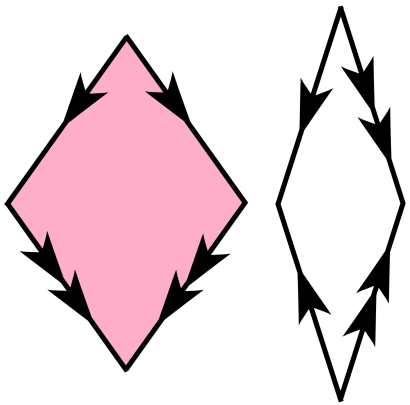
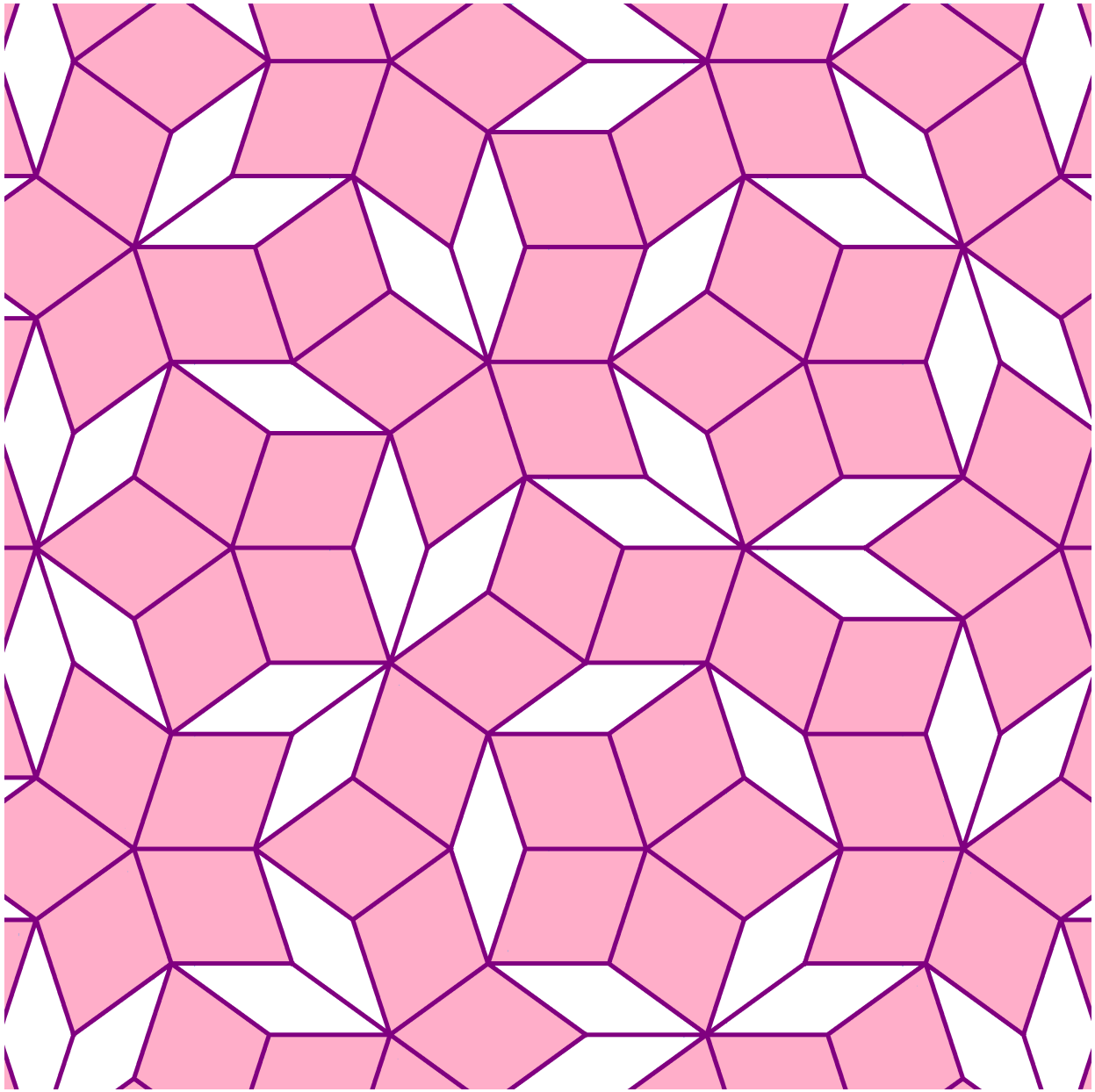
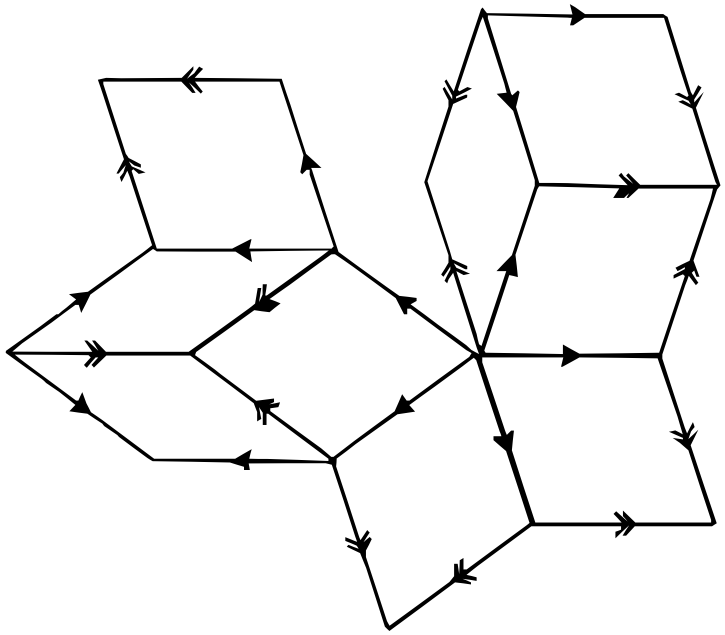
Penrose Tilings





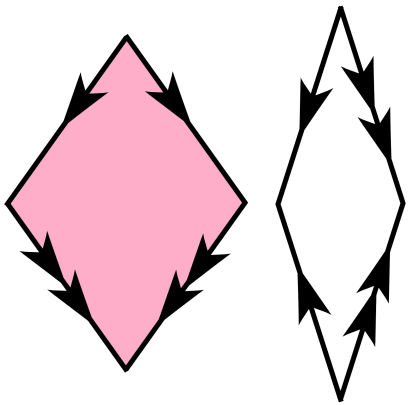
Local Rules and Global Ambiguities

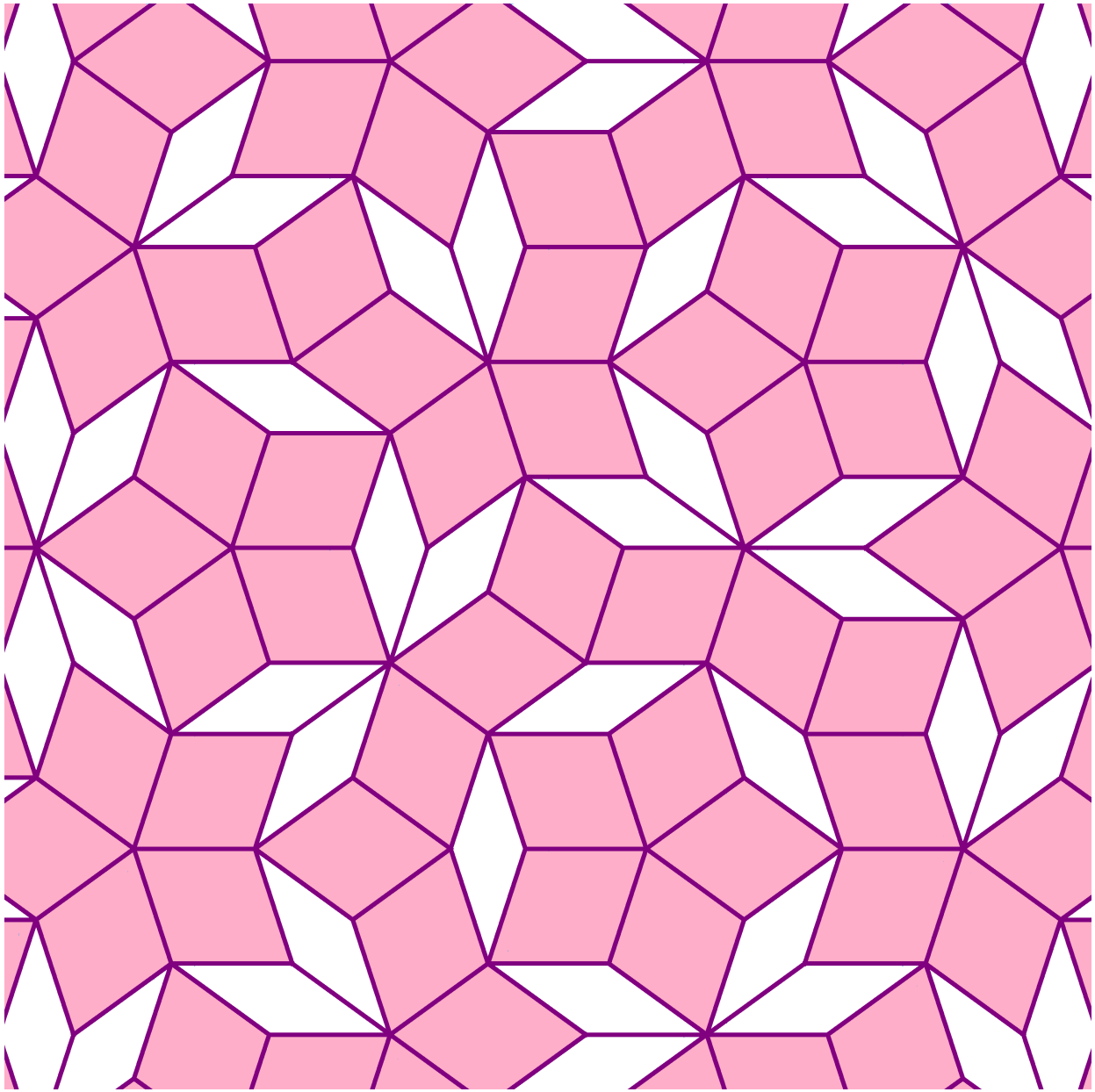
- **Quasicrystals** have **classically forbidden** symmetry patterns e.g. 5-fold and 10-fold symmetry in the Penrose Tiling
- Condensed matter physics, discrete holography, computability theory/logic, non-commutative geometry
- Assign **charges** to arrows: any simply connected patch has 0 net charge (since each tile has 0 net charge).



- Local matching rules can lead to ambiguities or errors.
  - No tile can fill this hole.
  - Patch cannot extend to a tiling of $\mathbb{R}^2$.

Penrose Tilings





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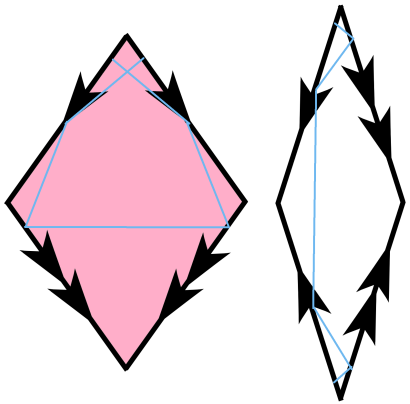

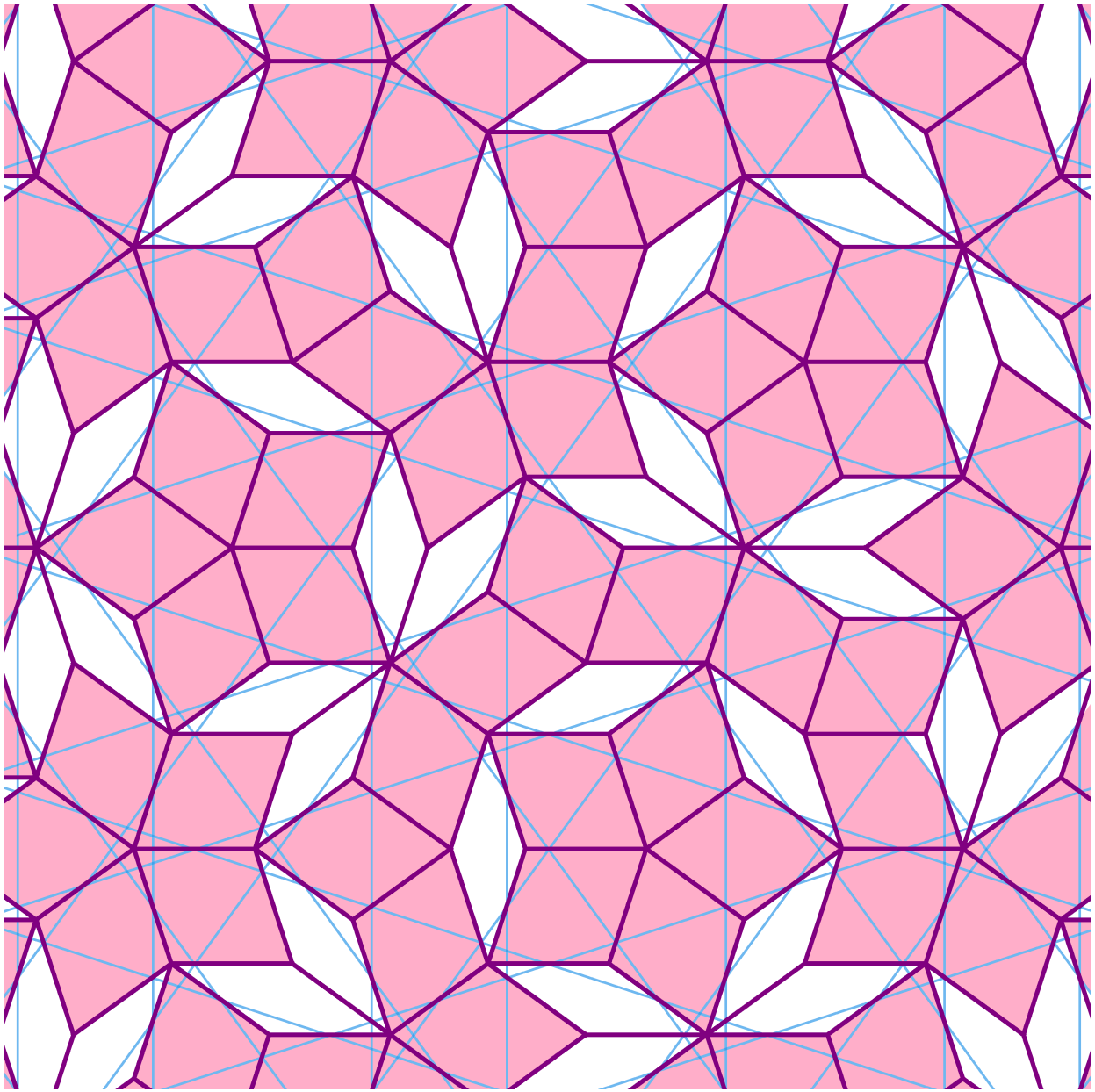

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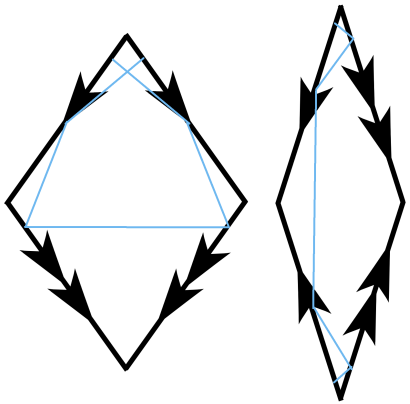
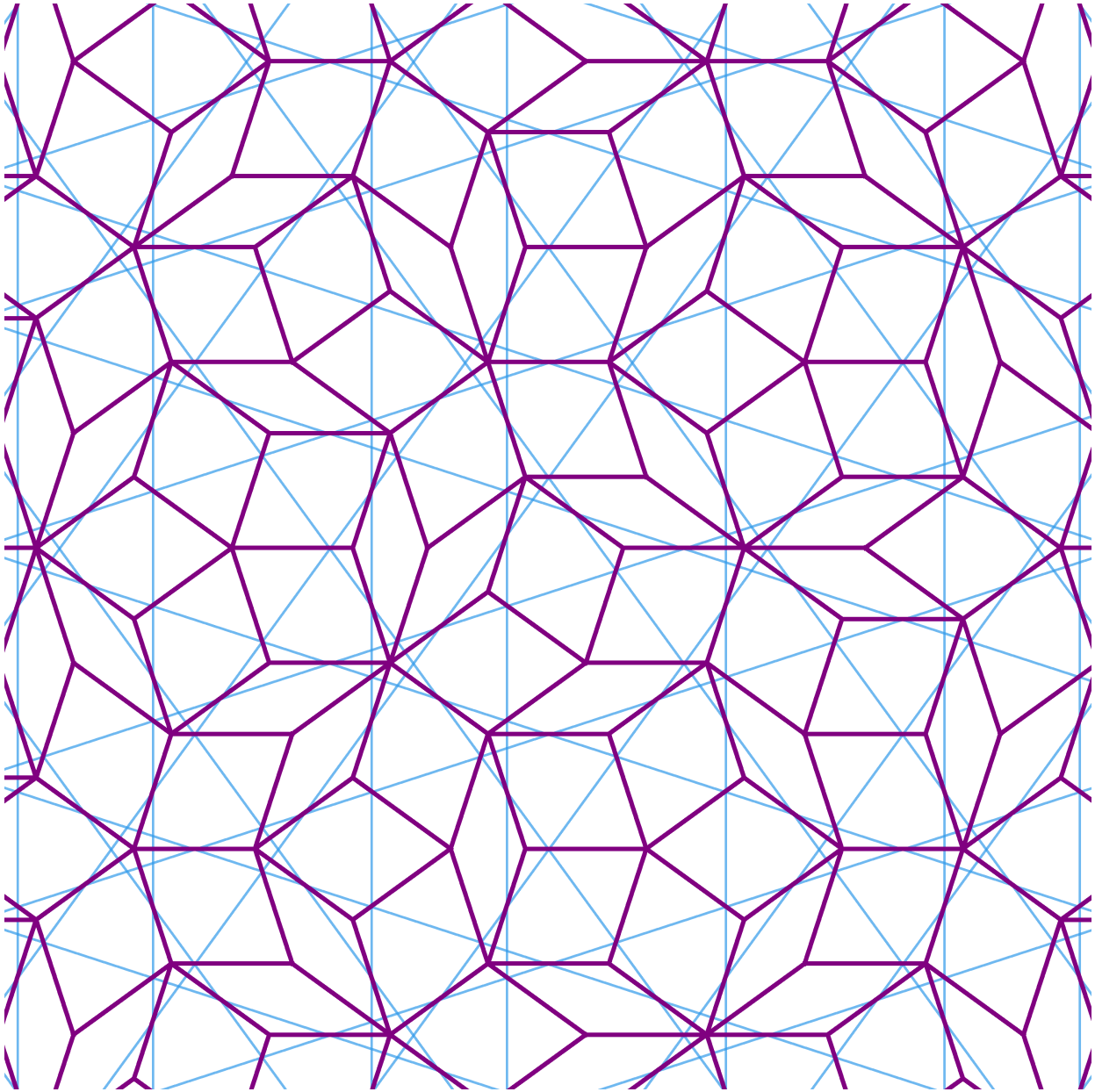
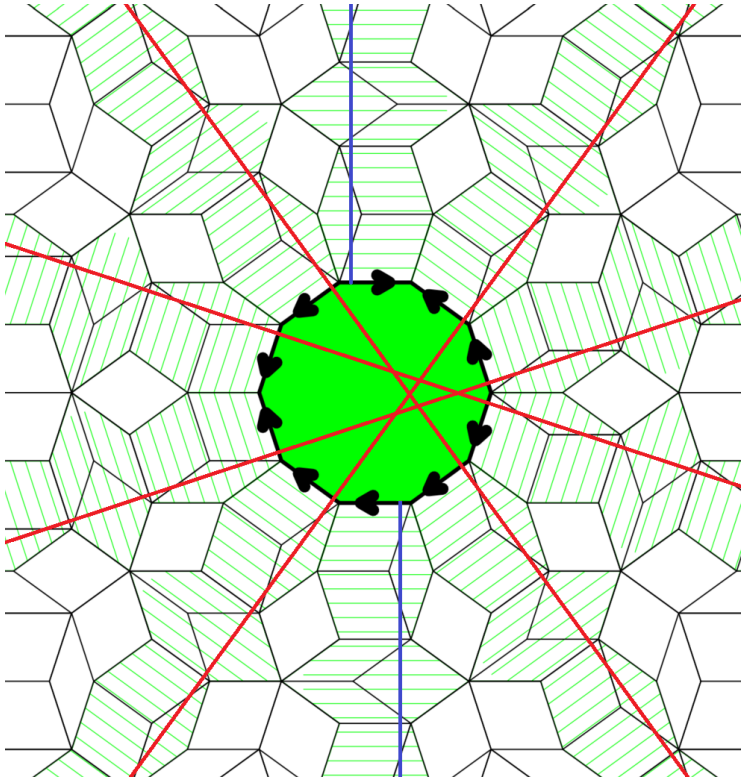




Conway's Decapods

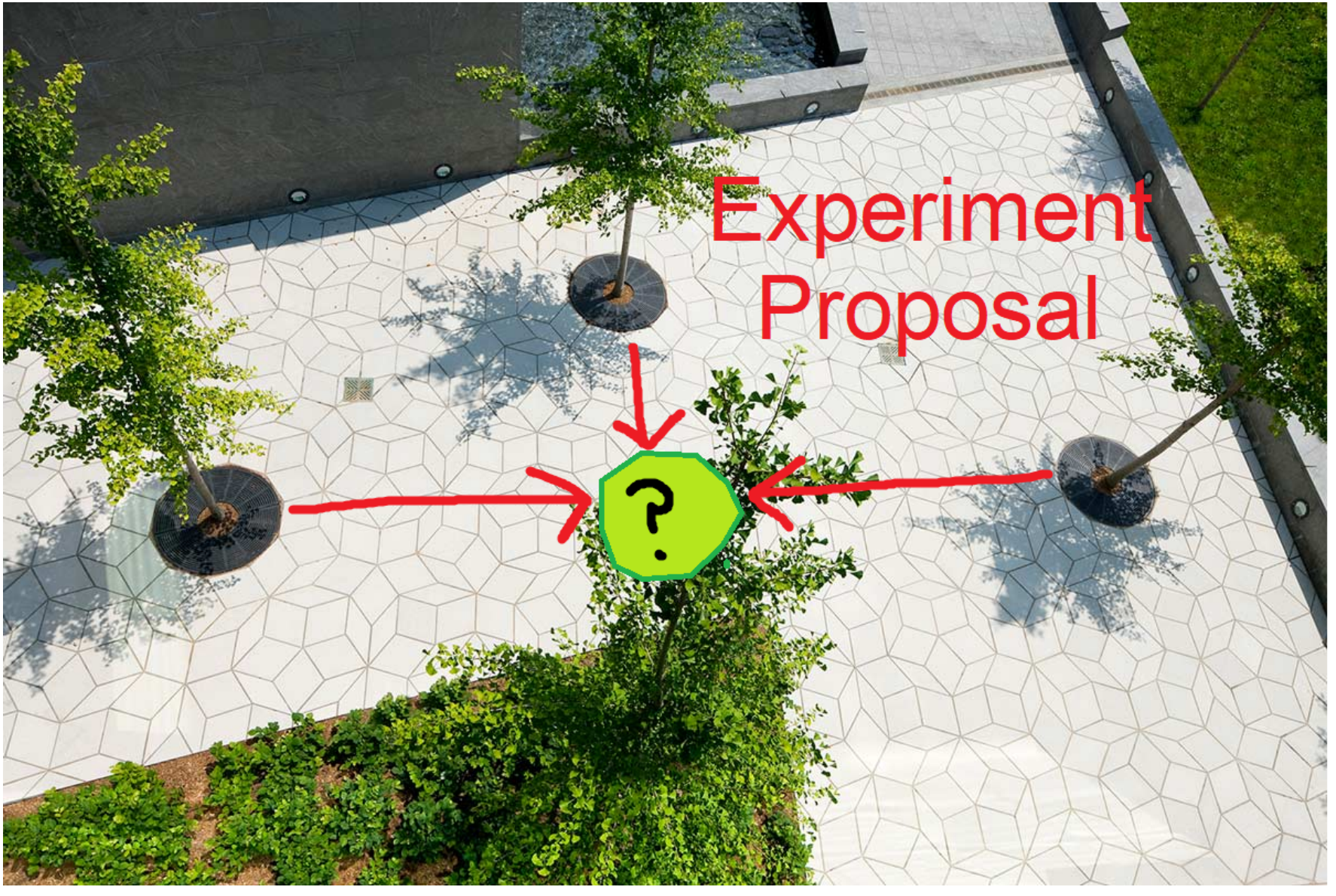
Conway's Decapods are defected: decagonal hole with spokes.

- No continuum description
- Decapods may **carry non-trivial charge**
- Ammann lines which do not match (blue) across the hole.



Conway's Defect Conjecture

Every possible hole is equivalent to a decapod hole by re-arranging a finite number of tiles around the hole.


Experiment
Proposal
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