Holomorphic Confinement of $\mathcal{N} = 1$ SYM

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Holomorphic Twists and Infinite Symmetries

- Given any SQFT, we obtain the **Holomorphic Twist** by taking cohomology of any nilpotent supercharge, e.g. $Q := Q_-$.
  - Anti-holomorphic translations are Q-exact, so twisted theory is (cohomologically) holomorphic [Johansen], [Witten], [Costello]

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- Protected subsector of **semi-chiral operators**, i.e. $[Q, \mathcal{O}] = 0$
  - In SCFTs: includes those counted by **superconformal index**

$$\mathcal{I} = \mathrm{Tr}(-1)^F p^{j_1+j_2-r/2} q^{j_1-j_2-r/2} e^{-\beta\{Q_-, S^-\}} \quad (2)$$

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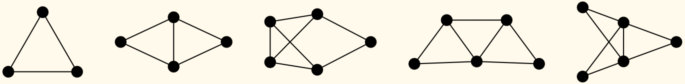
- Theories are equipped with local product called **λ-Bracket**

$$\{\mathcal{O}_1, \mathcal{O}_2\}_\lambda = \oint_{S^3} e^{\lambda\cdot z} d^2z \; \mathcal{O}_1(z)\, \mathcal{O}_2(0) \tag{3}$$

  - **Infinite dimensional symmetry enhancements** analogous to Virasoro and Kac-Moody, but in 4d [Gwilliam, Williams].
  - Higher brackets $\{\mathcal{O}_1, \mathcal{O}_2, \ldots, \mathcal{O}_{n+1}\}_{\lambda_1,\lambda_2,\ldots,\lambda_n}$

Cohomologies and Feynman Diagrams

- Feynman diagrams must be **Laman graphs**.



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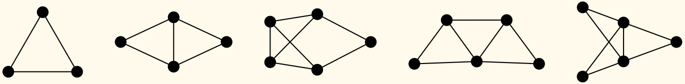

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- Polynomials in fields and derivatives $\rightsquigarrow$ **Free Cohomology** $\mathcal{V}$
  - **Interacting quantum theory** is obtained from underlying free-classical theory $\mathcal{V}$ as cohomology of a new operator

$$\mathbf{Q} = Q_0 + Q_1 + Q_2 \ldots . \tag{4}$$

    where Q_n is computed by n-loop Feynman diagrams.
  - All perturbative corrections are contained in the brackets!

$$\mathbf{Q}\,\mathcal{O} = \{\mathcal{I}, \mathcal{O}\}_0 + \{\mathcal{I}, \mathcal{I}, \mathcal{O}\}_{0,0} + \{\mathcal{I}, \mathcal{I}, \mathcal{I}, \mathcal{O}\}_{0,0,0} + \ldots . \tag{5}$$

Holomorphic Confinement of $\mathcal{N} = 1$ SYM

- $\mathcal{N} = 1$ **SYM** is $SU(N)$ gauge theory with an adjoint fermion

$$\mathcal{L} = \int d^2\theta \frac{-i}{8\pi} \tau \operatorname{tr} W_\alpha W^\alpha + \text{c.c.} \tag{6}$$

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- Being topological is compatible with **confinement**: if topological in the UV, then topological in the IR.
  - Constrains IR physics: the holomorphic twist of the IR must also be topological.